



# ENSTONE PRIMARY SCHOOL

## MATHEMATICS POLICY

### Aims

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

### Objectives

To enable children to:

- enjoy mathematics both as a tool for problem solving and for its own sake
- develop their thinking skills
- create opportunities for independent learning through teaching for creativity
- receive a broad and balanced curriculum which includes all aspects of mathematics (Number, Measurement, Geometry and Statistics) as well as a range of appropriate learning experiences (e.g. problem solving, practical work, games)
- use Digital Technology to develop their understanding of mathematical ideas and as a way of handling information effectively
- develop the numeracy skills they will need as foundation for further work when studying mathematics across the curriculum and in everyday life

### Teaching, Learning and Assessment

- This school uses the National Curriculum in England 2014 as the first basis for planning the teaching of mathematics.
- Cycles of coverage and medium term planning for all age groups is then based on the “White Rose” mathematics scheme of work.
- A mastery approach is used, as far as is possible within vertically grouped classes.

- A range of intervention and support strategies are used to ensure the needs of all learners are met. These include TA led 'catch up' groups, teacher led 'mop up' sessions for children who have had difficulty in understanding the concepts covered during the week's activities and one to one sessions for children who require extra support.
- Extension material is provided for more able children; additionally they may work alongside older children of a similar ability.
- Each aspect of mathematics is taught progressively throughout the school to ensure that skills and application are developed consistently.
- In Reception the daily mathematics lesson emphasises learning through stories, songs, games, imaginative play and practical activities so that the children enjoy using and experimenting with numbers and are given opportunities for effective mathematical development.
- Digital technology is used to support the teaching and learning of mathematics objectives, including *Abacus online* and *Purple Mash*. Calculators are introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure.
- Mathematics is applied and developed throughout the curriculum and in everyday life.
- Speaking and listening objectives are integral to mathematics. All classes adopt "talk partners" to facilitate this. Children are expected to explain their methods and strategies to others.

Assessment is an integral part of the teaching and learning process and is clearly linked to learning objectives:

- Learners are actively involved in self-assessment and reflection.
- Teachers' ongoing assessment informs planning.
- Mathematics tests (PUMA) are taken termly, and data is tracked such that intervention strategies can be planned where necessary to help children remain on track and achieve targets.

### **Equal Opportunities**

The full and effective participation and progression of all pupils should be achieved by:

- Ensuring access to learning at an individual level, through differentiated teaching and learning strategies, planning should set high expectations and provide appropriate learning opportunities for all pupils.
- Giving additional support as appropriate.
- Providing resources to support specific learning difficulties.

### **Resources**

- A range of resources to support the teaching and learning are kept in each classroom
- Some equipment, e.g. for measures, is held centrally
- Books and other published materials, are kept in the resource areas
- Online resources are available through whole school subscriptions

### **Calculation policy**

The development in calculation throughout the school is outlined in the calculation policy. We adhere to the calculation policy developed by "White Rose".

### **Developing Numeracy at home**

The school recognises the role of the family in supporting their children's mathematical development and aims to:

- provide children in years 1 to 6 with a weekly mathematics activity to do at home. This may be a game, puzzle, gathering data, learning number facts or consolidating and developing work done in class
- encourage parents to play a part in this activity, helping their child to become numerate
- develop parent's understanding of how maths is taught in school
- provide help and support to parents e.g. through open evenings and individual conversations between parents and teachers

**Date Adopted:** June 2023

**Review Date:** June 2026

**Signature of Chair of Governors:**

*Devin Caswell*

**Signature of Headteacher:**

*S. Mann*



# ENSTONE PRIMARY SCHOOL

## Calculation Policy

### ***Introduction***

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

**At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.**

**The overall aim is that when children leave primary school they:**

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

### **Mental methods of calculation**

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

**Secure mental calculation requires the ability to:**

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication and division facts up to  $10 \times 10$  (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2).
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

## **Written methods of calculation**

***The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient method for each operation with confidence and understanding. The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.***

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

## Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and **one** efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

**To add successfully, children need to be able to:**

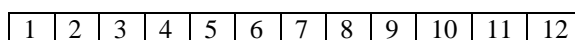
- recall all addition pairs to  $9 + 9$  and complements in 10, (such as  $\square + 3 = 10$ );
- add mentally a series of one-digit numbers, (such as  $5 + 8 + 4$ );
- add multiples of 10 (such as  $60 + 70$ ) or of 100, (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.**

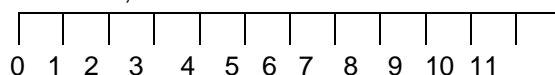
### Progression in use of number line

To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to be progression in their use of number tracks and number lines

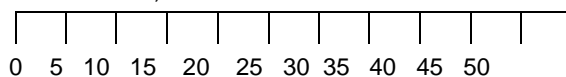
### Number track



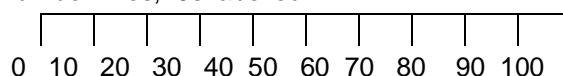
Number line, all numbers labelled



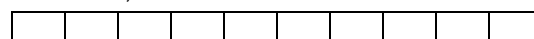
Number line, 5s and 10s labelled



Number lines, 10s labelled






Number lines, marked but unlabelled



Empty number line



<p><b>Stage 1: The empty number line</b></p> <ul style="list-style-type: none"> <li>The mental methods that lead to column addition generally involve partitioning. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</li> <li>The empty number line helps to record the steps on the way to calculating the total.</li> </ul>	<p><b>Stage 1</b></p> <p>Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.</p> <p><math>8 + 7 = 15</math></p>  <p><math>48 + 36 = 84</math></p>  <p>or:</p> 
<p><b>Stage 2: Partitioning</b></p> <ul style="list-style-type: none"> <li>The next stage is to record mental methods using partitioning into tens and ones separately. Add the tens and then the ones to form partial sums and then add these partial sums.</li> <li>Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.</li> </ul>	<p><b>Stage 2</b></p> <p>Record steps in addition using partitioning:</p> <p><math>47 + 76</math>  <math>47 + 70 + 6 = 117</math>  <math>117 + 6 = 123</math></p> <p>or <math>47 + 76</math>  <math>40 + 70 = 110</math>  <math>7 + 6 = 13</math>  <math>110 + 13 = 123</math></p> <p>Partitioned numbers are then written under one another, for example :</p> $\begin{array}{r} 47 = 40 + 7 \\ + 76 \quad 70 + 6 \\ \hline 110 + 13 = 123 \end{array}$
<p><b>Stage 3: Expanded method in columns</b></p> <ul style="list-style-type: none"> <li>Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums initially the tens, not the ones, are added first, following mental methods. The total of the partial sums can be found by adding them together.</li> <li>The addition of the tens in the calculation <math>47 + 76</math> is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. As children gain confidence, ask them to start by adding the ones digits first every time.</li> <li>The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.</li> </ul>	<p><b>Stage 3</b></p> <p>Write the numbers in columns.</p> <p>Adding the tens first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$ <p>Adding the ones first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$ <p>Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.</p>

**Stage 4: Compact column method**

- In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
- Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

**Stage 4**

$$\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array} \qquad \begin{array}{r} 366 \\ + 458 \\ \hline 824 \\ 11 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

## Written methods for subtraction of whole numbers

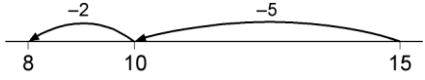
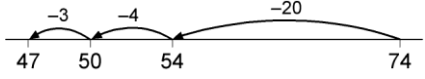
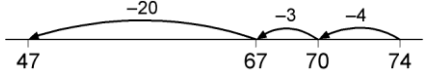
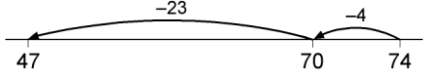
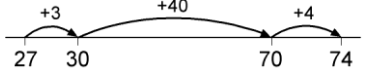
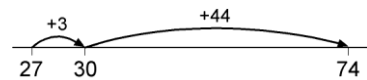
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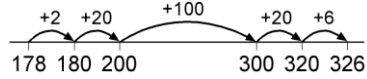
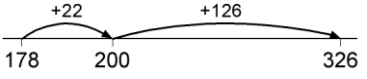
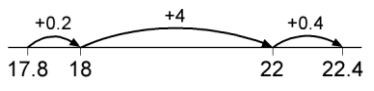
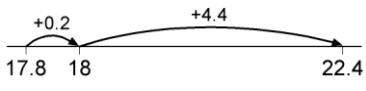
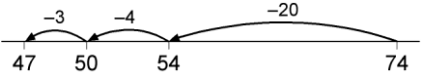
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers.

**To subtract successfully, children need to be able to:**

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as  $160 - 70$ ) using the related subtraction fact,  $16 - 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into  $70 + 4$  or  $60 + 14$ ).

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.**

<p><b>Stage 1: Using the empty number line</b>  <b>Finding an answer by counting back</b></p> <ul style="list-style-type: none"> <li>• The empty number line helps to record or explain the steps in mental subtraction.</li> <li>• A calculation like <math>74 - 27</math> can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.</li> </ul>	<p><b>Stage 1</b>  Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.  <math>15 - 7 = 8</math></p>  <p><math>74 - 27 = 47</math> worked by counting back:</p>  <p>The steps may be recorded in a different order:</p>  <p>or combined:</p> 
<p><b>Stage 1: Using an empty number line</b>  <b>Finding an answer by counting up</b></p> <ul style="list-style-type: none"> <li>• The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeepers method).</li> <li>• <b>With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as <math>57 - 12</math>, <math>86 - 77</math> or <math>43 - 28</math>.</b></li> </ul>	<p><math>74 - 27 =</math></p>  <p>or:</p> 

<ul style="list-style-type: none"> <li>With three-digit numbers the number of steps can again be reduced, enabling children to work out answers to calculations such as <math>326 - 178</math> first in small steps and then more compact by using knowledge of complements to 100</li> <li>The most compact form of recording becomes reasonably efficient.</li> </ul>	$326 - 178 =$  or: 
<ul style="list-style-type: none"> <li>The method can successfully be used with decimal numbers.</li> <li><b><i>This method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4 or below.</i></b></li> </ul>	$22.4 - 17.8 =$  or: 
<p><b>Stage 2: Partitioning</b></p> <ul style="list-style-type: none"> <li>Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For <math>74 - 27</math> this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn.</li> </ul> <p><b>This use of partitioning is a useful step towards the most commonly used column method, decomposition</b></p>	<p><b>Stage 2</b></p> <p>Subtraction can be recorded using partitioning:  <math>74 - 27</math>  <math>74 - 20 = 54</math>  <math>54 - 7 = 47</math></p> <p>This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.</p> 
<p><b>Stage 3:</b></p>	
<p><b>Expanded layout, leading to column method (Decomposition)</b></p> <ul style="list-style-type: none"> <li>Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.</li> <li>This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.</li> <li><b>The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.</b></li> </ul>	



**Stage 4: Compact method for three-digit numbers****NB Expanded method needs to be shown alongside compact method**

Example:  $563 - 241$ , no adjustment or decomposition needed

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 40 + 1 \\ \hline 300 + 20 + 2 \\ \hline \end{array} \qquad \begin{array}{r} 563 \\ - 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'

Example:  $563 - 246$ , adjustment from the tens to the units

$$\begin{array}{r} \begin{array}{r} 50 \quad 13 \\ 500 + \cancel{60} + \cancel{3} \\ - 200 + 40 + 6 \\ \hline 300 + 10 + 7 = 317 \end{array} \qquad \begin{array}{r} 51 \\ \cancel{563} \\ \underline{246} \\ 317 \end{array} \end{array}$$

Ensure that children can explain the compact method, referring to the real value of the digits. They need to understand that they are repartitioning the  $60 + 3$  as  $50 + 13$ .

Example:  $563 - 271$ , adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} \begin{array}{r} 400 \quad 160 \\ \cancel{500} + \cancel{60} + 3 \\ - 200 + 70 + 1 \\ \hline 200 \quad 90 \quad 2 = 292 \end{array} \qquad \begin{array}{r} 41 \\ \cancel{563} \\ \underline{271} \\ 292 \end{array} \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how  $500 + 60$  can be partitioned into  $400 + 160$ . The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Ensure that children are confident to explain how the numbers are repartitioned and why

## Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and

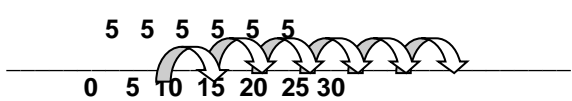
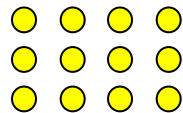
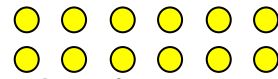
**One** efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

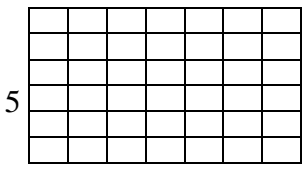
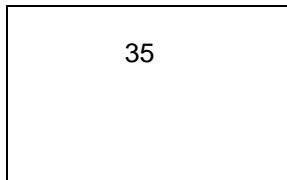
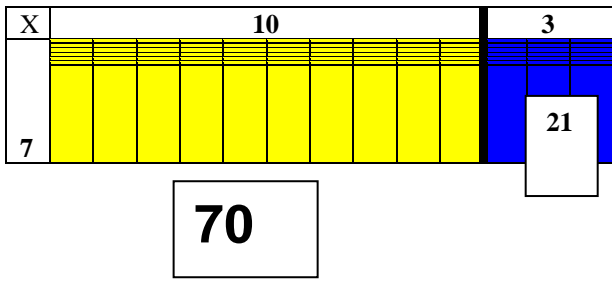
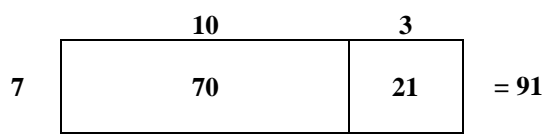
**To multiply successfully, children need to be able to:**

- recall all multiplication facts to  $10 \times 10$ ;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.**

### Developing the mental image of multiplication

<p><b>Stage 1 - Number lines</b></p> <p>This model illustrates how multiplication relates to repeated addition</p> <p>Pattern work on a 100 square helps children begin to recognise multiples and rules of divisibility</p>	<p><math>6 \times 5 =</math></p> 
<p><b>Arrays</b></p> <p>Successful written methods depend on visualising multiplication as a rectangular array. It also helps children to understand why <math>3 \times 4 = 4 \times 3</math></p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p><b>3 lots of 4</b> <math>3 \times 4</math></p>  <p><b>4 lots of 3</b> <math>4 \times 3</math></p> </div> <div style="text-align: center;"> <p><b>6 lots of 2</b> <math>6 \times 2</math></p>  <p><b>2 lots of 6</b> <math>2 \times 6</math></p> </div> </div>	

<p>The rectangular array gives a good visual model for multiplication. The area can be found by repeated addition (in this case <math>7+7+7+7+7</math>)</p> <p>Children should then commit <math>7 \times 5</math> to memory and know that it is the same as <math>5 \times 7</math></p>	<p><math>7 \times 5 = 35</math></p> <p style="text-align: center;">7</p> 
<p>Area models like this discourage the use of repeated addition. The focus is on the multiplication facts</p>	<p><math>7 \times 5 = 35</math></p> <p style="text-align: center;">7</p> 
<p><b>Stage 2</b> : Mental multiplication using arrays and partitioning to multiply a two-digit number by a one-digit number</p> <p>An array illustrates the distributive law of multiplication i.e.</p> <p><math>13 \times 7</math> is the same as <math>(10 \times 7) + (3 \times 7)</math></p> <p>Please note that the squares are used to ensure that children have a secure mental image of why the distributive law works</p>	<p><b><math>13 \times 7</math></b></p> 
<p>This can lead to the use of a "blank rectangle" to illustrate</p> <p><math>13 \times 7 = (10 \times 7) + (3 \times 7)</math></p> <p><b>Note the rectangle is drawn to emphasise the comparative size of the numbers</b></p>	<p><b><math>13 \times 7</math></b></p> 

<p>Using the grid method to multiply two-digit by one-digit numbers</p> <p>At first children will probably need to partition into 10's (example A)</p> <p>It is important, if they are to use a more compact method, that they can multiply multiples of 10 (example B)</p> <p>i.e. <math>38 \times 7</math> they must be able to calculate <math>30 \times 7</math> as well as <math>8 \times 7</math></p> <p><b>Note the grid is drawn to emphasise the comparative size of the numbers</b></p>	<p><b><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></b></p> <p><b>Example A</b></p> <table border="1" style="margin-left: 40px;"> <tr> <td></td> <td style="text-align: center;">10</td> <td style="text-align: center;">10</td> <td style="text-align: center;">10</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">7</td> <td style="text-align: center;">70</td> <td style="text-align: center;">70</td> <td style="text-align: center;">70</td> <td style="text-align: center;">56</td> <td></td> </tr> </table> <p><b>Example B</b></p> <table border="1" style="margin-left: 40px;"> <tr> <td></td> <td style="text-align: center;">30</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">7</td> <td style="text-align: center;">210</td> <td style="text-align: center;">56</td> <td style="text-align: right;">=266</td> </tr> </table> <p><b>Leading to the layout</b></p> <table border="1" style="margin-left: 40px;"> <tr> <td style="text-align: right;">X</td> <td style="text-align: center;">30</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">7</td> <td style="text-align: center;">210</td> <td style="text-align: center;">56</td> <td style="text-align: right;">= 266</td> </tr> </table> <p><b>This will lead to a more formalised layout</b></p>		10	10	10	8		7	70	70	70	56			30	8		7	210	56	=266	X	30	8		7	210	56	= 266
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<p><b>Stage 3:</b> Two-digit by two-digit products using the grid method</p> <p>Extend to <math>TU \times TU</math>, asking children to estimate first.</p> <p>Start by completing the grid. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product</p> <p><b>Please note that at this stage the grid is no longer drawn to reflect the respective size of the digits. If a child shows signs of insecurity return to rectangular arrays to ensure understanding</b></p>	<p><b>Stage 3</b></p> <p><math>38 \times 14</math> is approximately <math>40 \times 15 = 600</math>.</p> <table border="1" style="margin-left: 40px;"> <tr> <td style="text-align: right;">X</td> <td style="text-align: center;">30</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">10</td> <td style="text-align: center;">300</td> <td style="text-align: center;">80</td> <td style="text-align: right;">380</td> </tr> <tr> <td style="text-align: right;">4</td> <td style="text-align: center;">120</td> <td style="text-align: center;">32</td> <td style="text-align: right;">152</td> </tr> <tr> <td></td> <td></td> <td></td> <td style="text-align: right;">532</td> </tr> </table>	X	30	8		10	300	80	380	4	120	32	152				532												
X	30	8																											
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<p>Three-digit by two-digit products using the grid method</p> <p>Extend to <math>HTU \times TU</math> asking children to estimate first.</p> <p><b>Ensure that children can explain why this method works and where the numbers and the grid come from</b></p>	<p><b><math>138 \times 24 =</math> is approximately <math>140 \times 25 = 3500</math></b></p> <table border="1" style="margin-left: 40px;"> <tr> <td style="text-align: right;">X</td> <td style="text-align: center;">100</td> <td style="text-align: center;">30</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">20</td> <td style="text-align: center;">2000</td> <td style="text-align: center;">600</td> <td style="text-align: center;">160</td> <td style="text-align: right;">2760</td> </tr> <tr> <td style="text-align: right;">4</td> <td style="text-align: center;">400</td> <td style="text-align: center;">120</td> <td style="text-align: center;">32</td> <td style="text-align: right;">552</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td style="text-align: right;">3312</td> </tr> </table>	X	100	30	8		20	2000	600	160	2760	4	400	120	32	552					3312								
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<p>The grid method works just as satisfactorily with decimal numbers as long as the children can apply their knowledge of multiplication facts to decimal numbers.</p>	<p><b><math>38.5 \times 24</math> is approximately <math>40 \times 25 = 1000</math></b></p> <table border="1" style="margin-left: 40px;"> <tr> <td style="text-align: right;">X</td> <td style="text-align: center;">30</td> <td style="text-align: center;">8</td> <td style="text-align: center;">0.5</td> <td></td> </tr> <tr> <td style="text-align: right;">20</td> <td style="text-align: center;">600</td> <td style="text-align: center;">160</td> <td style="text-align: center;">10</td> <td style="text-align: right;">770</td> </tr> <tr> <td style="text-align: right;">4</td> <td style="text-align: center;">120</td> <td style="text-align: center;">32</td> <td style="text-align: center;">2</td> <td style="text-align: right;">154</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td style="text-align: right;">924</td> </tr> </table>	X	30	8	0.5		20	600	160	10	770	4	120	32	2	154					924								
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<p><b>Optional Stage 4: Expanded short multiplication leading to column method</b></p> <ul style="list-style-type: none"> <li>The first step is to represent the method of recording in a column format but showing the working. Draw attention to the links with the grid method above.</li> <li>Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in <math>38 \times 7</math> is 'thirty multiplied by seven', not 'three times seven', although the relationship <math>3 \times 7</math> should be stressed.</li> </ul>	<p><b>Stage 4</b></p> <p><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></p> $\begin{array}{r} 30 + 8 \\ \times \quad 7 \\ \hline 210 \quad 30 \times 7 \\ \underline{56} \quad 8 \times 7 \\ \hline 266 \end{array}$ $\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 210 \\ \underline{56} \\ \hline 266 \end{array}$
<p><b>Short multiplication</b></p> <ul style="list-style-type: none"> <li>The recording is reduced further, with carry digits recorded below the line.</li> </ul> <p><b>If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of the grid method</b></p>	<p><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></p> $\begin{array}{r} 38 \\ \times \quad 7 \\ \hline \underline{266} \\ 5 \end{array}$ <p>The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.</p>
<ul style="list-style-type: none"> <li>Multiplying two-digit by two-digit numbers includes the working to emphasise the link to the grid method</li> </ul>	<p><math>56 \times 27</math> is approximately <math>60 \times 30 = 1800</math>.</p> $\begin{array}{r} 56 \\ \times \quad 27 \\ \hline 1000 \quad 50 \times 20 = 1000 \\ 120 \quad 6 \times 20 = 120 \\ 350 \quad 50 \times 7 = 350 \\ \underline{42} \quad 6 \times 7 = 42 \\ \hline 1512 \\ 1 \end{array}$
<p><b>Three-digit by two-digit numbers</b></p> <ul style="list-style-type: none"> <li>Continue to show working to link to the grid method.</li> <li>This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive for more confident children to move on to a more compact method.</li> </ul>	$\begin{array}{r} 286 \\ \times \quad 29 \\ \hline 4000 \quad 200 \times 20 = 4000 \\ 1600 \quad 80 \times 20 = 1600 \\ 120 \quad 6 \times 20 = 120 \\ 1800 \quad 200 \times 9 = 1800 \\ 720 \quad 80 \times 9 = 720 \\ \underline{54} \quad 6 \times 9 = 54 \\ \hline 8294 \\ 1 \end{array}$
<p><b>Note most primary school children are unlikely to be ready to use any method more compact by the end of year 6</b></p>	

## Written methods for division of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and **one** efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

**To divide successfully in their heads, children need to be able to:**

- understand and use the vocabulary of division
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to  $10 \times 10$ , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.**

**To carry out written methods of division successfully, children also need to be able to:**

- understand division as repeated subtraction (Grouping):
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- Know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g.  $120 - 80$ ,  $320 - 90$

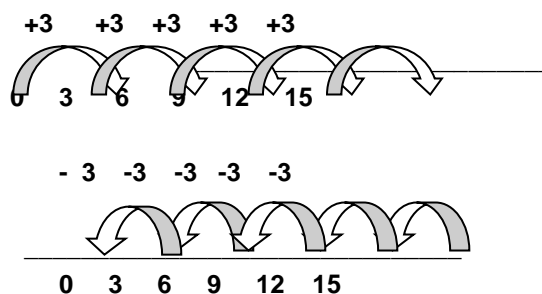
### Stage 1 Number lines


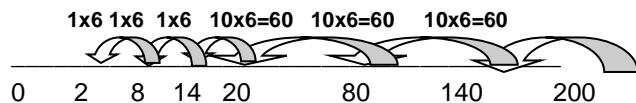
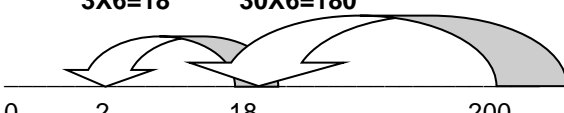
Counting on in equal steps based on adding multiples up to the number to be divided

Counting back in equal steps based on subtracting multiples from the number to be divided

**Note** Counting on is a powerful tool for mental calculation but does not lead onto written calculation for division

$$15 \div 3 =$$



<p><b>Stage 2 Counting back by chunking</b></p> <p>This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. Chunking is useful for reminding children of the link between division and repeated subtraction.</p>	<p><math>100 \div 7 =</math></p> <p><b><math>4 \times 7 = 28</math></b>      <b><math>10 \times 7 = 70</math></b></p>  <p>Answer 14 remainder 2</p> <p>As you record the division, ask: 'How many sixes in 100?' as well as 'What is 100 divided by 6?'</p>																								
<p>Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.</p> <p><b>Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples</b></p>	<p><math>200 \div 6</math></p>  <p>Answer 33 remainder 2</p> <p>As you record the division, ask: 'How many sixes in 200?' as well as 'What is 200 divided by 6?'</p> <p>Leading to</p> <p><math>200 \div 6</math></p> <p><b><math>3 \times 6 = 18</math></b>      <b><math>30 \times 6 = 180</math></b></p> 																								
<p><b>'Expanded' method for <math>TU \div U</math> recorded in columns</b></p> <ul style="list-style-type: none"> <li>This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.</li> <li>As you record the division, ask: 'How many sixes in 90?' or 'What is 90 divided by 6?'</li> <li>This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.</li> <li>Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps as illustrated in stage 2, when using a number line</li> </ul>	<p><math>96 \div 6 =</math></p> <p>To find <math>96 \div 6</math>, we start by multiplying 6 by 10, to find that <math>6 \times 10 = 60</math> and <math>6 \times 20 = 120</math>. The multiples of 60 and 120 trap the number 96. This tells us that the answer to <math>196 \div 6</math> is between 60 and 120.</p> <p>Start the division by first subtracting 60 leaving 36, and then subtracting the largest possible multiple of 6, which is 30, leaving no remainder.</p> <table style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td></td><td style="text-align: right;">96</td><td></td></tr> <tr><td style="text-align: right;">-</td><td style="text-align: right;">60</td><td style="text-align: right;">10 X 6</td></tr> <tr><td></td><td style="text-align: right;">36</td><td></td></tr> <tr><td></td><td style="text-align: right;">30</td><td style="text-align: right;">5 X 6</td></tr> <tr><td></td><td style="text-align: right;">6</td><td></td></tr> <tr><td></td><td style="text-align: right;">6</td><td style="text-align: right;">1 X 6</td></tr> <tr><td></td><td style="text-align: right;">0</td><td style="text-align: right;">16</td></tr> <tr><td></td><td colspan="2" style="text-align: center;">Answer 16</td></tr> </tbody> </table> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin-left: auto; margin-right: auto;"> <p><b>Estimation</b> More than <math>10 \times 6 = 60</math> but less than <math>20 \times 6 = 120</math></p> </div>		96		-	60	10 X 6		36			30	5 X 6		6			6	1 X 6		0	16		Answer 16	
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**Stage 3 : 'Expanded' method for HTU ÷ U**

- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.

The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.

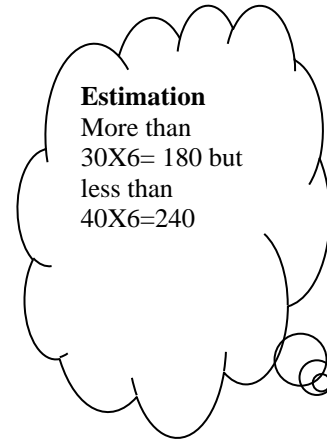
- Estimating has two purposes when doing a division:
  - to help to choose a starting point for the division;
  - to check the answer after the calculation.

To find  $196 \div 6$ , we start by multiplying 6 by 10, 20, 30, to find that  $6 \times 30 = 180$  and  $6 \times 40 = 240$ . The multiples of 180 and 240 trap the number 196. This tells us that the answer to  $196 \div 6$  is between 30 and 40.

Initially children will subtract chunks about which they are totally confident. Here a series of chunks ( $6 \times 10$ ) are subtracted to reach 16 then  $6 \times 2$  until no more whole sixes are left, leaving a remainder of 4

$$\begin{array}{r}
 196 \\
 - \quad 60 \quad 10 \times 6 \\
 \hline
 136 \\
 \quad 60 \quad 10 \times 6 \\
 \hline
 76 \\
 \quad 60 \quad 10 \times 6 \\
 \hline
 16 \\
 \quad 12 \quad 2 \times 6 \\
 \hline
 4 \quad 32
 \end{array}$$

Answer 32 R 4



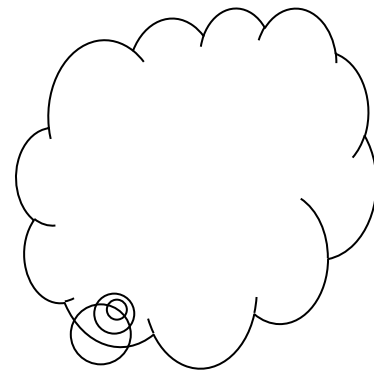
- Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

Here the child has been confident to use the largest possible multiple of 10 as the initial multiplier.

Start the division by first subtracting 180 ( $6 \times 30$ ), leaving 16 and then subtracting the largest possible multiple of 6 (which is 12) leaving 4

$$\begin{array}{r}
 196 \\
 - \quad 180 \quad 30 \times 6 \\
 \hline
 16 \\
 - \quad 12 \quad 2 \times 6 \\
 \hline
 4 \quad 32
 \end{array}$$

Answer 32 R 4



The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

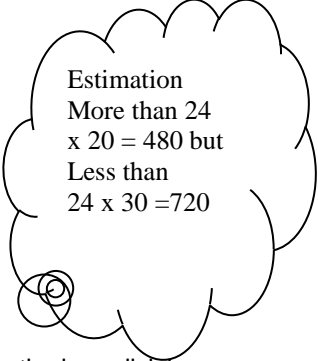
### Long division

The next step is to tackle  $HTU \div TU$ , which for most children will be in Year 6.

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As  $24 \times 20 = 480$  and  $24 \times 30 = 720$ , we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{r} 24 \overline{) 560} \\ 20 \underline{- 480} \quad 24 \times 20 \\ \quad 80 \\ 3 \quad \underline{72} \quad 24 \times 3 \\ \quad \quad 8 \end{array}$$

Answer: 23 R 8



Estimation  
More than 24  
 $\times 20 = 480$  but  
Less than  
 $24 \times 30 = 720$

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} \quad 23 \\ 24 \overline{) 560} \\ \underline{-480} \\ \quad 80 \\ \underline{-72} \\ \quad \quad 8 \end{array}$$

Answer: 23 R 8